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ABSTRACT

In recognition of the fact that young children develop informal number concepts and strategies for solving several basic types of verbal addition and subtraction problems before entering school, but lose this ability after a few years of schooling, the study created and tested an intervention designed to build on the children's early insights. A plan was developed for initial instruction in verbal problem solving that would measure the effects of teaching children to symbolically represent verbal problems with number sentences that directly modeled verbal problem structures. The plan was based on a review of concrete and symbolic representation in mathematics and canonical or noncanonical categories of number sentences to represent various categories of verbal addition and subtraction problems. A pretest treatment, posttest design was used. The subjects (N=45) were two first-grade classes in a rural area outside Madison, Wisconsin. Evaluation of the pretest-posttest differences showed that the children made significant gains both in solving verbal problems and in writing correct number sentences. Appendices include the problem classification scheme, problem form or type, actual pretest and posttest, and analyzed results. A five-page reference list is included. (JM)

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CHILDREN'S SYMBOLIC REPRESENTATION OF ADDITION AND SUBTRACTION VERBAL PROBLEMS

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Introduction

Before entering school, young children have developed informal number concepts and strategies for solving several types of basic addition and subtraction verbal problems (see summaries in Carpenter, Blume, Hiebert, Anick, & Pimm, 1982; Ginsburg, 1977; Starkey & Gelman, 1982). An important feature of children's informal solution strategies is their apparent understanding of problem structure, evidenced by their attempts to represent problem structure with concrete items (Blume, 1981; Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, 1982, 1983, 1984; Hiebert, 1982). Prior to formal instruction, children set out concrete items to represent the verbal problem and then use these concrete representations to find the solution.

These informal insights into verbal problem structure are not evident in children's later solution attempts after a few years of formal schooling. National assessment results indicate that by the age of nine, many children seem to solve verbal problems with a more superficial approach. They appear to extract the given numbers, assign an operation, and state an answer (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981). These later performances indicate a lack of insight into problem structure, an insight that appeared to be present in earlier verbal problem solution attempts.

The problem then that this study addresses is that young children

before formal instruction can solve several types of verbal problems with concrete models that represent problem structure, but that at a later age and after formal instruction children attempt to solve verbal problems by attending mainly to superficial aspects of the verbal problem. A shift away from early successful informal strategies to later less successful strategies appears to occur in many children.

The occurrence of early insights into verbal problem structure suggests a direction for initial formal instruction in verbal problem solving. Initial instruction might be planned to focus and build on these informal problem insights that children bring to school. Specifically, initial formal problem-solving instruction should capitalize on children's informal abilities to understand and represent problem structure.

To investigate such a possibility, this study developed a plan for initial instruction in verbal problem solving that taught children to symbolically represent verbal problems with number sentences that directly represented verbal problem structure. The study intended to extend research on children's informal solutions of verbal problems into a plan for formal problem-solving instruction.

This paper is presented in three major parts. The first presents representation of verbal problems as a general topic in mathematics education, and concrete representation and symbolic representation as specific topics. The second part describes the research study. The final part relates the findings to possible directions for problem-solving instruction.

REPRESENTATION OF VERBAL PROBLEMS

The topic of representation has been discussed both in terms of general problem solving and in terms of mathematics verbal problems. Bruner (1964) singled out representation along with integration as the two most important components of intellectual or cognitive growth. Representation is essential, stated Bruner, because "children, as they grow, must acquire ways of representing the recurrent regularities in their environment" (p. 13).

In the area of general problem solving, Paige and Simon (1966) emphasized auxiliary representation as an important component in problem solving processes of humans as opposed to direct translation, defined by them as a word-by-word interpretation. They stated that the auxiliary representation of a problem is derived from a solver's prior knowledge and forms a link between the physical conditions of the problem and the mathematics equation that can be used for the solution. Auxiliary representation was inferred as being a pictorial or graphical representation of the problem that occurs prior to the formal symbolic mathematical representation.

In a position paper related to mathematics learning, Romberg (1975) described representation as one of the five "intellectual processes" that an individual uses to solve problems. Romberg's statement about representation included the concept of auxiliary representation:

> "The representation processes allow the student to progress from solving problems directly to solving abstractly. Students begin by solving problems directly with the objects or sets involved. In solving many problems, they gradually learn to use physical, then pictorial, and finally symbolic representations to help them" (p. 164).

Romberg ascribed to representation processes the purpose of linking the physical problem with mathematics symbols.

Weaver (1982) advocated a type of auxiliary representation system in suggesting that children be taught a form of symbolism prior to formal mathematics symbolism. He hypothesized that "there is a distinct advantage in using a mediating notational form prior to... conventional number sentence or equation..." (pp. 65-66). His specific instructional suggestions included either an arrow diagram, such as a $\xrightarrow{+b}$ c, or a flowchart-type diagram.

Vergnaud (1982) addressed the topic of auxiliary symbolic representation for verbal problems as part of a recommended "psychogenetic" approach to mathematics learning. He suggested five types of symbolic systems as auxiliary representations intended to illuminate the structure of verbal problems: four of these systems were diagrams: distance, vector, transformation, and Euler-Venn, and the fifth was an algebraic equation form. Vergnaud proposed that efficient symbolic representations should help students both to differentiate among various types of problems and to solve problems that they would otherwise be unable to solve. He suggested that to best promote learning on the part of the child, one must know which of the symbolic representations are the easiest, the most naturally used, and the most easily assimilated by children.

Several other researchers in mathematics education have advocated some type of representation of the verbal problem, either concrete or symbolic, as being beneficial to young learners' solution performances. Steffe (1968) was a proponent of an "interpretation" or "intermediate step" between the verbal problem statement and the solution.

> "There may be a step missing in this sequence teaching to the number fact, in that the children are rarely asked to give an interpretation (using objects or otherwise) of an addition problem verbally related to them by the teacher. This [interpretation] certainly may be a desirable intermediate step in the route to the number fact and should be explored further" (Steffe, 1968, p. 51).

Carpenter (1985) indicated the importance of representation to verbal problem solving in relating young children's concrete models for solving verbal problems to representation. He stated that, "At the most basic level, problem solving reduces to a question of finding an appropriate representation for a problem so that it can be solved" (p. 39).

Lindvall and Ibarra (1980a) reviewed studies indicating that "effective problem solvers make use of an intermediate stage between their initial comprehension of the problem and their development of a mathematical equation of solving it. In this stage, they abstract the essential information and develop some type of conceptual representation of the problem, mental or physical" (p. 4). Lindvall and Ibarra maintained that by emphasizing an intermediate step of representation, students could not only solve verbal problems more successfully, but could also acquire a valuable strategy to be used in other problem-solving situations.

DeCorte and Verschaffel (1983) also supported an initial problem representation step prior to computational solution. These authors suggested that the current instruction on problem solving supports the selection and performance of arithmetic operations and excludes the important problem-solving step of representation.

This distinction between problem representation and problem solution, as referred to by Lindvall and Ibarra (1980a) and DeCorte and Verschaffel (1983), may be difficult to observe. With simple problems that involve recall of fact knowledge, the answer appears to be immediately available and a representation strategy seems unnecessary. With more difficult problems for which an answer is not readily available, a strategy of representation may be identified in the process of obtaining the answer.

Representation of verbal problems may be considered as concrete representations for which children have manipulatives available to help them solve the problems, or as symbolic representations for which children are asked to write a number sentence to solve a verbal problem. The following sections discuss the topic of representation from these two viewpoints.

Concrete Representation

Information about children's success rates in solving verbal mathematics problems with concrete items available is provided by mathematics education and cognitive psychology research (Briars & Larkin, 1984; Carpenter & Moser, 1983; Riley, Greeno, & Heller, 1983). Findings indicate that children initially solve verbal problems with concrete representations or models, and that these models reflect the underlying mathematical structure of certain problems (Blume, 1981; Carpenter et al, 1981; Carpenter & Moser 1982, 1983, 1984; Hiebert, 1982).

From data on children's concrete modeling strategies and success rates, addition and subtraction verbal problems have been classified according to problem type and ordered according to problem difficulty. Tables 1 and 2 present the current classification of addition and subtraction verbal problems (Carpenter & Moser, 1983; Riley et al., 1983).

Classifications of Verbal Problems.

Table 1 displays the four classifications of verbal addition and subtraction problems determined by the inclusion of either an action or a static situation variable. Change and Equalize problems involve action situations whereas Combine and Compare types involve static situations. Table 1 also separates Change and Combine problems into two categories, addition or Join categories and subtraction or Separate

categories.

(Insert Table 1)

A further delineation of the Change problems is based on the position of the unknown element in the structure-based mathematical equation, or number sentence. Table 2 presents the six Change problems with corresponding number sentence forms and types that directly represent the various positions the unknown can take to reflect the underlying structure of these problems.

(Insert Table 2)

As displayed in Table 2, the mathematical structure of Change 1 and 2 problems can be directly represented by the respective canonical sentences A + B = ☐ and A - B = ☐. Canonical forms of number sentences are those "typical" equations in which the unknown, displayed here as an empty box, appears alone on either side of the equals sign. In the present discussion, canonical sentences are limited to those in which the unknown appears to the right of the equals sign.

Table 2 also indicates that the mathematical structure of Change 3, 4, 5, and 6 problems can be directly represented by the noncanonical sentences A + ☐ = C, A - ☐ = C, ☐ + B = C, and ☐ - B = C, respectively. Noncanonical forms of number sentences are those less "typical" equations in which the unknown appears along with one or more numerals on either side of the equals sign. The noncanonical sentences in this discussion are limited to those in which the unknown appears to the left of the equals sign along with one numeral.

As indicated also in Table 2, Change problems and related open number sentence forms are referred to as Result Unknowns, Change Unknowns, and Start Unknowns, according to the position of the unknown in the corresponding number sentence. The Result Unknown problems are

represented by number sentences that are canonical; in other words, the result of the number sentence is unknown, as in the following sentences A + B = ☐ or A - B = ☐. The Change Unknown and Start Unknown problems are represented by number sentences that are noncanonical. Change Unknowns are of the form A + ☐ = C or A - ☐ = C, forms in which the numerical change occurring in the sentence is unknown. Start Unknowns are of the form ☐ + B = C or ☐ - B = C, forms in which the starting number of the sentence is unknown.

Concrete Representational Strategies.

At the most basic level of solution, children use a variety of concrete representational strategies to represent and to solve verbal problems. Although a variety of solution strategies are used by children, certain of the verbal problem types are solved by most children with specific concrete representational strategies. Table 3 presents a summary of children's most common concrete representational strategies corresponding to each problem type (Carpenter & Moser, 1984). These strategies exhibit the most basic concrete representations used by children to solve verbal problems, and involve direct representation of the structure of the verbal problems with concrete items.

(Insert Table 3)

Children appear to move through levels of verbal problem solving performance. Models of young children's progressive levels of solution strategies have been proposed by Briars and Larkin (1984), Carpenter and Moser (1984), and Riley and others (1983). These models were based on young children's concrete representations of various types of verbal problems. Differences among these three models were discussed by Carpenter (1985).

Studies Documenting Children's Concrete Representation of Verbal Problems.

Researchers have documented children's success in solving certain types of addition and subtraction verbal problems (Blume, 1981; Carpenter et al., 1981; Carpenter & Moser, 1983; DeCorte, Verschaffel, & DeWin, 1984; Hiebert, 1982; Lindvall & Ibarra, 1980b; Riley, et al., 1983; Steffe, 1970; Steffe & Johnson, 1971; Tamburino, 1980). Studies by Steffe (1970) and by Steffe and Johnson (1971) were among the earliest studies to indicate that with concrete items available for representation, first-graders were successful in solving several types of verbal problems.

Other studies provide information on various problem types. Most studies that included both Change 2 and Combine 2 subtraction problems problems showed substantial success differences between these two problems. These differences indicate children's success in more accurately representing the Change 2 action situation than the static Combine 2 (Carpenter et al., 1981; Carpenter & Moser, 1983; Lindvall & Ibarra, 1979; Riley et al., 1983; Tamburino, 1980).

Most studies that included both types of Change Unknown problems, Change 3 and 4, found considerably better results for Change 4 types (Hiebert, 1982; Riley et al., 1983; Tamburino, 1980). Hiebert (1982) discussed the possible fortuitous rather than insightful choice of strategies for Change 4 problems.

Most studies indicated that children are less successful solving Start Unknown problems, Change 5 and 6, than other types (DeCorte et al., 1984; Hiebert, 1982; Riley et al., 1983; Tamburino, 1980). Hiebert (1982) suggested a hierarchy of increasing verbal problem difficulty from Result Unknown to Change Unknown to Start Unknown based on children's ease in concretely representing problems. Lindvall and

Ibarra's studies (1979; 1980b) presented higher results than other studies on the Start Unknown problems, especially with the optimal mode of the examiner concretely representing the problem for the child.

Blume's study (1981) documented that children's concrete models directly represented the structure of the verbal problem. He demonstrated that children used distinctly different strategies for Change 2 and Change 3 problems, specifically a Separating From strategy for Change 2 and an Adding On strategy for Change 3.

Evidence from these studies of children's concrete representations of verbal problems indicate that for the most part children are successful in solving verbal problems with concrete items available for representation. Concrete models become unwieldy as the problem number domain increases and appear to be replaced by counting sequence strategies (Carpenter & Moser, 1984). In these counting strategies, children use numerals instead of concrete items to represent verbal problem givens. Children's departures from concrete representations to counting strategies indicate possible attempts to progress away from concrete methods of solution to methods that involve the symbols of mathematics.

Symbolic Representation

A traditional goal of instruction in mathematics has been to teach children to use the symbols of mathematics to represent problems from the physical world. Van Engen (1949) viewed the purpose of initial instruction in mathematics as supporting such a move from the concrete to the symbolic. Implying that initial instruction should prepare students for later study in mathematics, Van Engen advocated that children learn to use the symbols of mathematics to represent their concrete experiences: "The whole object of arithmetic instruction clearly is to help the child devise a system of symbols which, in some

sense, is representative of a realm of events--a series of symbolized operations with which the child has had direct experiences" (1949, p. 325).

Studies Documenting Children's Symbolic Representation of Verbal Problems.

Recent research studies have investigated children's success in writing number sentences to symbolically represent and solve addition and subtraction verbal problems (Carpenter, Bebout, & Moser, 1985; Carpenter, Hiebert, & Moser, 1983; DeCorte & Verschaffel, 1984; Lindvall & Ibarra, 1980b; Moser & Carpenter, 1982). Results for Change and Combine problems from these five studies are presented in Table 4. In these studies children were asked to write number sentences for various types of verbal problems.

(Insert Table 4)

Lindvall and Ibarra (1980b) studied the attempts of 81 first-graders to write number sentences for Change 3, 4, 5, and 6 and Combine 1 problems. Over 60% of the children were successful in writing sentences for Change 3, Change 5, and Combine 1 problems, and about 45% were successful in writing sentences for the Change 4 and Change 6 problems. The authors determined, from their investigation of other related problem tasks, that the tasks of concretely modeling and of solving a verbal problem were prerequisites to the task of writing a correct number sentence.

Carpenter and others (1983) reported results of a study with 43 first-graders. Children were asked to write number sentences for Change 2, Combine 1, Combine 2, Compare, and Equalize verbal problems after instruction that was based on a part/whole analysis of verbal problems. Children were successful in writing correct number sentences

for the Change and Combine problems and less successful for the Compare and Equalize problems.

Moser and Carpenter (1982) taught a pilot group of four first-graders to write number sentences that directly modeled the structure of Change 1, 2, 3, 6, and Combine 1 problems with a microcomputer program. This pilot group was successful in learning to write structure-based number sentences.

Carpenter and others (1985) instructed first- and second-graders in writing number sentences for Change 1, 2, and 3 problems. Posttest results for the 11 first-graders are presented in Table 4. These results indicated various levels of success in writing number sentences for the noninstructed problems, Change 4, 5, 6, and Combine 1 and 2, as well as for the instructed problems, Change 1, 2, and 3.

DeCorte and Verschaffel (DeCorte & Verschaffel, 1983; Verschaffel, 1984) studied 30 first-graders who were receiving instruction in "finding the number sentence 'hidden' in the verbal text" (Verschaffel, 1984, p. 10). They reported that correct sentences were written by these children on over 75% of the Change 1, 2, and 3 and Combine 1 and 2 problems, and on over 62% of the Change 6 problems.

Besides reporting correct sentence data, DeCorte and Verschaffel also indicated the number sentence forms that children used to symbolically represent verbal problems. They stated that these number sentence forms represented either (1) the structure of the verbal problem or (2) the arithmetic operation necessary to solve the verbal problem (DeCorte, Verschaffel, Janssens, & Joillet, 1984). This distinction between these two number sentence forms can be illustrated by considering the number sentence forms that symbolically represent the following Change 3 problem:

Polly had 3 cookies.
Joe gave her some more cookies.
Now she has 7 cookies.
How many cookies did Joe give her?

A number sentence to symbolically represent the structure of this problem is 3 + ☐ = 7; a number sentence to symbolically represent the arithmetic operation necessary for solution is 7 - 3 = ☐. While both sentences result in the correct answer, the sentence representing the structure more nearly matches children's informal concrete modeling strategy for this problem as reported by Blume (1981). When children solve this problem with concrete items, they set out 3 items, add on items to the original set until the final set of 7 is reached, and then count the number of items added on to obtain the solution of 4.

A number sentence to symbolically represent the arithmetic operation, 7 - 3 = ☐, may be difficult for young children to produce because the correspondence between the concrete model and the number sentence may not be readily apparent. In order to write such a number sentence, a child may have to transform or rerepresent the verbal problem. Briars and Larkin (1984) suggest that such a transformation, away from direct representation of the problem structure to a rerepresentation for the arithmetic operation, may require the use of advanced mathematical abilities beyond those of many young children.

An unanswered question in the mathematics education literature has concerned a related issue, the relative effectiveness of teaching children to write number sentences that symbolically represent either the verbal problem structure or the arithmetic operation necessary for solution (Hartung, 1959; 1962; Van Engen, 1959). A study carried out by Wilson (1967) with fourth-graders indicated that children at that age were more successful in writing number sentences that represented the arithmetic operation than in writing number sentences that

represented the problem structure. Zweng (1968), in a reaction to this study, stated that number sentences that symbolically represented problem structure were more in keeping with the major objective of mathematics instruction, that of "translation of the physical world into mathematical models" (1968, p. 253). Recently, Weaver (personal communication, November, 1984) surmised that writing sentences to symbolically represent the arithmetic operation was probably more similar to the fourth-graders' prior learning experiences than were sentences to symbolically represent problem structure.

Three studies in Table 4 reported results that indicated the form of number sentences that children wrote to symbolically represent various verbal problems. These were studies by Moser and Carpenter (1982), DeCorte and Verschaffel (1983), and Carpenter and others (1985). These studies indicated that many first-graders attempted to use problem structure as the basis for their number sentences.

Canonical and Noncanonical Number Sentence Forms.

The use of verbal problem structure as the basis for symbolic representation results in noncanonical forms of number sentences for Change Unknown problems (Change 3 and 4) and Start Unknown problems (Change 5 and 6), as well as canonical forms for Result Unknown problems (Change 1 and 2). Noncanonical forms have had the reputation of being more difficult for children to solve than canonical forms. This reputation is somewhat difficult to substantiate. Conversely, certain studies from research on children's success in solving abstract problems or open sentences, i.e., number sentences containing an unknown, indicate that children are fairly successful in solving both canonical and noncanonical forms of open sentences (Blume, 1981; Grouws, 1972; Weaver, 1971; 1973; summary in Carpenter et al, 1982). Table 5 displays data from studies that investigated children's

solution performance on six open sentence forms, two canonical and four noncanonical forms. The data indicate that, although children are generally more successful in solving canonical forms of open sentences, their performance on noncanonical forms is not inadequate. In fact, their performance on noncanonical forms might be considered remarkable if the suspected low instructional exposure is taken into account (Weaver, 1971).

(Insert Table 5)

Summary

This section has presented a background for the representation of verbal problems. Studies that focused on children's success in concretely representing and solving verbal problems have been presented. Other studies have been presented to document children's attempts to symbolically represent verbal problems. The study described in the following section was based on children's success in concretely representing verbal problem structure. An instructional treatment was developed to teach the children to write number sentences to symbolically represent verbal problem structure. Symbolically representing problem structure required the children to use noncanonical forms for certain problems and canonical forms for other problems. Children's abilities to understand problem structure, as indicated by their concrete representations, provided the rationale for the instructional treatment.

Research Study

Method

This study investigated the effects of teaching children to symbolically represent verbal problems with number sentences that directly modeled problem structure. Children were evaluated before instruction with verbal problem tests that asked them to write a number sentence for and to solve certain verbal problems. Instruction followed that intended to teach them to symbolically represent the structure of verbal problems with canonical or noncanonical number sentences. After instruction, children's performances were reevaluated with verbal problem tests that asked them to write a number sentence for and to solve certain verbal problems.

The sample consisted of 45 first-graders in two classrooms during the spring months. The school was located in a rural community outside of Madison, Wisconsin.

Instructional Content.

On the basis that prior to instruction children successfully solve verbal problems by attending to and concretely modeling problem structure, instruction began with and reinforced these concrete models. This reinforcement was followed by replacement of the concrete models with number sentences consisting of the formal symbols of mathematics. These symbols were the numerals 0-11, the operational and relational signs of plus, minus, and equals, and an empty box to indicate the unknown. Following reinforcement of the concrete model and replacement with a number sentence, instruction focused on teaching the children to write a number sentence without the concrete modeling step. Instruction intended to connect the verbal problems with number

sentences by teaching students not only to write number sentences to represent verbal problems, but also the reverse, by teaching students to compose verbal problems to match given number sentences (Van Engen & Grouws, 1975).

Along with direct instruction on symbolically representing verbal problems with number sentences, indirect instruction occurred on solving the resultant open number sentences. Students were encouraged to solve the number sentences using their choice of concrete modeling, counting, or number fact strategies.

A summary of instructional steps is indicated in Table 6. Instruction for each verbal problem type proceeded according to the following steps: (1) concrete representation of the verbal problem to obtain a solution, (2) symbolic representation of the verbal problem to replace the concrete representation, (3) symbolic representation directly from the verbal problem without the concrete representation, and (4) composition of a verbal problem to match a given number sentence.

(Insert Table 6)

Based on the current classification of verbal problems (see Table 1), two major types of problems, Change and Combine, were used for instruction. The order of introduction of the verbal problem types was from the easiest to the most difficult as determined by previous research on verbal and abstract problem solving (Carpenter et al., 1982; Hiebert, 1982; Weaver, 1971). This hierarchy is displayed in Table 7. Instruction began with the Result Unknown types, first Change 1 and 2 and then Combine 1 and 2. Instruction followed with the Change Unknown types, Change 3 and 4, and ended with the Start Unknown types, Change 5 and 6.

(Insert Table 7)

Each of the four pairs of Change and Combine problem types displayed in Table 7 were taught according to the instructional steps presented in Table 6. The 14 instructional lessons developed the four-step instructional plan for each verbal problem pair and included a cumulative review of previously presented pairs.[1]

Instructional Procedures.

Instruction was presented in the existing classroom settings during the 45-minute mathematics period three times a week for five weeks by the author of this study, an experienced elementary school teacher.

The organization of the classroom for instruction followed the plan for effective teaching set forth by Good, Grouws, and Ebmeier (1983) according to these authors' major lesson parts of review, development, and practice.

The review part of the lesson was planned to be brief and quickly paced, to bring the learners' attention to a common content point, to involve the less assertive students, and to provide a common basis for new content planned for the development lesson part that followed. The main purpose of the development portion of the daily lesson was to lead the class as a whole to an understanding of intended content. This was carried out by whole-class instructional episodes and materials. Development time varied and was adjusted to provide maximum whole-class instruction as determined by the students' attention. Important features of the practice portion of the lesson were that the duration

[1] Detailed lesson plans with accompanying activity sheets are available from the author.

was fifteen minutes or less, the settings were student pairs or small groups, the materials were individual activity sheets, the teaching episodes were private, and feedback was immediate from student responses displayed on the blackboard.

Evaluation.

The primary instruments of evaluation consisted of ten verbal problems administered before and after instruction (see Tables 8 and 9). In these preinstructional and postinstructional evaluation situations, each child was given a booklet of ten verbal problems and was asked both to write a number sentence for and to solve each problem. A tester read the verbal problems to the students. No manipulatives were available during the testing situations.

(Insert Tables 8 & 9)

Coding of the tests focused on two features of the students' written performances, the number sentence for and the answer to the verbal problem. The number sentences were coded as to form and correctness. The answers were coded as correct or incorrect.

Three forms of number sentences were coded as follows: Direct Representation, Rerepresentation, and Other. The Direct Representation category included those number sentence forms that symbolically represented the verbal problem structure. These forms were further coded as Complete or Incomplete, depending on mathematical correctness. The Rerepresentation category included those number sentence forms that symbolically represented the arithmetic operation necessary for solution. These forms too were further classified as Complete and Incomplete. The Other category included those number sentences that symbolically represented neither the problem structure nor the arithmetic solution operation. An Other category was also coded if a

student made no attempt to write a number sentence.

To be considered in the Complete rather than the Incomplete category, a number sentence had to contain the following: (1) the two numbers stated in the verbal problem, (2) an operational sign that correctly related the two stated numbers, (3) an equals sign positioned correctly, and (4) an element to indicate the answer or unknown in the form of an empty box, a numeral, or even a space.

An answer to a verbal problem was termed Correct if the student's written number sentence included the correct numeral regardless of position in the sentence or other symbols present. If a student made no attempt to solve the verbal problem, an Incorrect category was coded.

Results

Data for children's symbolic representations and solutions of verbal problems before and after instruction are presented in Tables 10 and 11. Data for Change problems are presented in Table 10, and data for Combine, Compare, and Equalize problems are presented in Table 11.

(Insert Tables 10 & 11)

Verbal Problem Solution.

Prior to instruction nearly 60% of the children were successful in symbolically solving Change 1, 2, 4, and Combine 1 problems. A large percentage could not solve the other problem types. Eighty percent could not solve the Change 5 problem, over 70% could not solve the Combine 2 and Compare problem, and over 60% could not solve the Change 3, Change 6, and Equalize problems.

After instruction, a larger percentage of children were successful in symbolically solving all problem types. Over 75% were successful in solving the Result Unknown problems (Change 1 and 2 and Combine 1 and

2) and the addition Change Unknown and Start Unknown problems (Change 3 and Change 5). Over 60 % were successful in solving the subtraction Change Unknown and Start Unknown problems (Change 4 and Change 6) and the Equalize problem. Over 50% were successful in solving the Compare problem.

A comparison of children's pretest and posttest performances in solving verbal problems symbolically is displayed in Table 12. This comparison indicates that children made significant gains in symbolically solving the instructed verbal problem types.

(Insert Table 12)

Verbal Problem Representation.

Data for children's symbolic representation of verbal problems are also presented in Tables 10 and 11. For each problem type, the total number of correct number sentences and the forms of number sentences are displayed.

Before instruction, nearly 70% of the children were successful in writing correct number sentences for Change 1, Change 2, and Combine 1 problems, and over 40% were successful for the Change 4 problem. For these four problem types, the children wrote correct canonical number sentences.

A large percentage of children were not successful in writing correct number sentences for the other problem types before instruction. Over 90% were unsuccessful on Change 3, Change 5, Compare, and Equalize problems. Nearly 80% were unsuccessful on Change 6 and Combine 2 problems.

The forms of number sentences that children wrote before instruction for Change Unknown and Start Unknown problems are interesting. Some children attempted to directly represent problem

structure with their number sentences. Over 13% attempted to directly represent the structure of Change 3, 4, 5, and 6 problems with noncanonical number sentences. These attempts are indicated in Table 10 as Incomplete Direct Representation sentences. Some children attempted to transform or rerepresent the problems. Sixty percent rerepresented the Change 4 problem and nearly 25% rerepresented the Change 6 problem. These rerepresentations were canonical number sentence forms and were categorized as both Complete and Incomplete Rerepresentations.

Posttest results indicated that after instruction nearly 80% of the children were successful in writing correct number sentences for the eight Change and Combine problems, the instructed problem types. The forms of number sentences written after instruction differed from those written before instruction for the Change Unknown and Start Unknown problems. For Change 3, 5, and 6 problems, nearly 90% of the children wrote Direct Representation sentences, number sentences that directly represented the problem structure. For the Change 4 problem, nearly 80% wrote Direct Representation sentences and about 20% wrote Rerepresentation sentences.

A comparison of children's pretest and posttest performances in writing correct number sentences is displayed in Table 13. This comparison indicates that children made significant gains in symbolically representing the verbal problem types.

(Insert Table 13)

Summary.

For the noninstructed problems, Compare and Equalize, children were not successful in writing correct number sentences on the posttest. However, over half the children were successful in solving the

problems, and over 17% wrote Direct Representation sentences that were intended to directly represent problem structure after instruction.

In summary, after instruction, children were successful in symbolically representing and solving a larger set of problems. They were taught during instruction a means for symbolically representing the structure of verbal problems. They also appeared to have learned correct number sentence form during instruction.

Discussion

Results of this study pertain to a single instructional treatment; no comparison with other instructional treatments is performed by this study. Thus, results cannot be considered to be more or less effective than other instructional treatments might have been.

A major result of this instructional treatment was that over 80% of the children were successful in writing correct number sentences to symbolically represent eight types of Change and Combine problems, the instructed verbal problem types. A reason for their successes may have been that the children were taught to write number sentence forms that directly represented the structure of the verbal problems. These number sentence forms corresponded to their informal concrete representations that were based on problem structure.

Prior to instruction, children's exposure to number sentence forms was probably limited to canonical forms. Children can use canonical forms to directly represent the structure of Change 1, Change 2, and Combine 1 problems, but may not be able to relate canonical forms to the structures of other verbal problems. If children are taught only canonical forms of number sentences, they are not provided with the options that mathematical symbolism permits; they are not made aware that mathematical symbolism can be used to directly represent the

physical world. If children are limited to writing only canonical number sentence forms, they may not be able to make a connection between their informal concrete representations and the formal symbolic representations, or number sentences. With only canonical forms, the children are limited to symbolically representing only certain verbal problems or possibly problems that have already been solved (Carpenter et al., 1983).

The inclusion of noncanonical as well as canonical forms in initial number sentence writing instruction appeared to provide these children with the means of symbolically representing more types of verbal problems. Children were taught that the form of a number sentence can vary according to the structure of the verbal problem. The pretest sentence writing results in this study indicated that some children attempted to write noncanonical forms when asked to symbolically represent certain problems even though they probably had not had exposure to these forms.

Another reason for the children's increased success in writing correct and complete number sentences after instruction was probably the instructional emphasis on the inclusion of the necessary symbolic elements for accurate number sentences. Before writing number sentences, the children used symbol cards to lay out the necessary components of a number sentence, two numeral cards, an operation card, an equals sign card, and a card with an empty box drawn on it to signify the unknown. Early instruction emphasized that these five cards were all necessary elements. Later instruction emphasized using these necessary elements but varying their positions to reflect different problem structures.

An important number sentence element stressed by this

instructional plan was an empty box to represent the unknown element of the problem. In a study with kindergarteners, Lindvall and Ibarra (1979) noted the positive effect that a marker for the unknown had on young children's success in solving problems. The use of such a marker may have had a similar positive effect in this study on children's success in symbolically representing as well as solving verbal problems.

In fact, over 60% of the children in this study were successful in solving Start Unknown problems (Change 5 and 6). Their successes might be attributed to their having had a means for representing the unknown element that starts these problem types. Carpenter (1985) surmised that a reason for most children's lack of success in solving Start Unknown problems with concrete items was that children had no way to concretely model the starting unknown set. By learning the symbolic elements necessary for a complete number sentence, especially an empty box to signify the unknown, the children in this study may have found a means for representing the starting set in Start Unknown problems that assisted them in solving these problem types.

Another major result of this instructional treatment was that over 60% of the children were successful in solving the instructed verbal problem types symbolically. Although their successes could be partly attributed to the instructional exposure they received, another important factor could be that they learned a means of symbolically representing the problem prior to solution. Representation has been presented in the first section of this paper as an important step in problem solving. Posttest results from this study may further support representation as important prior to solution.

Children in this study received direct instruction in

symbolically representing each verbal problem prior to solution; the number sentence had to be stated before the answer. Symbolic representation was directly taught and was the focus of whole-class instruction, whereas the solution was indirectly taught and was performed by each child on an individual basis.

Limitations of the Study

A limitation of this study was that it provided only a single instructional treatment. A research design that included other plans for initial instruction in problem solving might provide more definitive results on the most suitable instruction for young children. Kouba and Moser (1979) planned instruction to teach first-graders to symbolically represent verbal problems with a part-whole analysis of each problem. Carpenter, Hiebert, and Moser's (1983) study looked at children's symbolic representations after instruction based on part-whole problem analysis. Riley, Greeno, and Heller's (1983) conception of children's analyses of verbal problems supports part-whole instruction. A future study that presents both types of instruction, symbolic representation of verbal problem structure and symbolic representation based on part-whole analysis, might provide more information on which plan is most suitable for initial instruction.

Another important limitation of this study was inherent in the research design that based pretest and posttest data on single responses from young children, cited as being flexible in their use of strategies (Carpenter & Moser, 1982, 1983). Data reliability would be enhanced by repeated measures of children's responses.

The research design of the present study was also limited by a lack of control over the number triplets used for the problem tasks. Although specific basic fact domains were used to assign number

triplets to verbal problems, information related to children's knowledge of particular facts might enhance this design.

A further limitation of this study was that no allowance was made for teacher characteristics. Although some variation was provided for classroom climate by including two separate classrooms in the sample, the same instructor taught both groups of children.

A final limitation of this study involved the noninclusion during instruction of the Compare and Equalize problem types. The posttest symbolic solution results indicate that many children were more successful in solving these problems symbolically, but that most children were less successful in symbolically representing these problems. An instructional plan that included these problem types would add to the information on all types of verbal problems.

Implications for Instruction.

Mathematics education is closer to establishing an instructional science for initial programs of mathematics instruction in addition and subtraction verbal problem solving. Research results from cognitive psychologists and mathematics educators have uncovered many of the mathematical strategies that young children use prior to formal instruction. Researchers suggest that initial formal instruction in mathematics that takes into account and builds on the informal abilities of the child might be most conducive to learning (Carpenter & Moser, 1984; Case & Bereiter, 1984; Vergnaud, 1982). By using what is currently known about children's informal mathematical strategies as a basis for formal initial instructional design, the establishment of research-based initial instruction in mathematics may be possible.

It is known that many children are able to solve addition and subtraction verbal problems before they come to school. The types of

problems that are easiest and most difficult are known also. The strategies that are used by children can be predicted too. A hierarchy of problem solving levels based on young children's successes and failures is being established. This knowledge about children's thinking now needs to be incorporated into the mathematics curriculum.

An important result from research on children's thinking has been the finding that children appear to understand and attempt to model the mathematical structure of verbal problems before they receive formal instruction. By capitalizing on this knowledge about children's informal understanding of problem structure, initial instruction in mathematics might link children's informal knowledge with the formal content area of mathematics. The link might be made by teaching children to use the symbols of mathematics to directly represent the verbal problem structure, in a manner that corresponds with their use of concrete models to represent the structure. Such a plan for initial instruction both capitalizes on the informal knowledge that children bring to instruction and connects this knowledge with the mathematical content area.

This study demonstrated a successful initial instructional program in mathematics that taught children to use the symbols of mathematics to match their informal understanding of verbal problem structure. They learned the essential elements of a number sentence, and they learned that they could vary the positions of these elements to match verbal problem structure. They learned this variance because they were taught the mathematical legitimacy of both canonical and noncanonical number sentence forms.

Such an instructional program does not suggest a superficial approach to teaching children to symbolically represent problems with a

key word approach or with a numbers-operator representation (Briars & Larkin, 1984). The basis for this instruction lies in children's understanding of the structure of the verbal problem and in their understanding of the essential elements of simple equations.

Unanswered questions exist as to the next instructional step after children are taught to symbolically represent problem structure with number sentences. Most children in this study were successful in solving the resultant canonical and noncanonical forms, but more information is needed on the best ways of instructing children to solve these forms. More information is needed too on the best time to teach children to transform their noncanonical sentences into canonical forms. Such an area leads into initial instruction in algebraic expressions. The answers to the best ways to organize the subsequent instruction are open to future curriculum research.

For now, initial instruction in mathematics might be planned that both capitalizes on children's informal knowledge of verbal problem solving and that provides children with the power of mathematical symbolism. Teaching children to symbolically represent verbal problems with canonical and noncanonical number sentences may provide the means of connecting children's informal knowledge with the formal content area of mathematics.

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Table 1

Classification of Verbal Problems

Change

Join	Separate
1. Polly had A cookies. Joe gave her B more cookies. How many cookies does Polly have now?	2. Polly had A cookies. She gave B cookies to Joe. How many cookies does Polly have left?
3. Polly had A cookies. Joe gave her some more cookies. Now she has C cookies. How many cookies did Joe give her?	4. Polly had A cookies. She gave some to Joe. Now she has C cookies. How many cookies did she give to Joe?
5. Polly had some cookies. Joe gave her B more cookies. Now she has C cookies. How many cookies did she have to start with?	6. Polly had some cookies. She gave B cookies to Joe. Now she has C cookies. How many cookies did she have to start with?

Combine

1. Polly has A red marbles and B blue marbles. How many marbles does she have in all?	2. Polly has A marbles. B marbles are red and the rest are blue. How many blue marbles does Polly have?

Compare

Polly has A marbles.
Joe has B marbles.
How many more marbles does Polly have than Joe?

Equalize

Polly has A marbles.
Joe has B marbles.
How many more marbles does Joe have to get to have as many as Polly?

Table 2

Change Problems with Corresponding Forms and Types of Number Sentences

Verbal Problem	Number Sentence Form	Type
1. Polly had A cookies. Joe gave her B more cookies. How many cookies does Polly have now?	A + B = ☐	Result Unknown
2. Polly had A cookies. She gave B cookies to Joe. How many cookies does Polly have left?	A - B = ☐	Result Unknown
3. Polly had A cookies. Joe gave her some more cookies. Now she has C cookies. How many cookies did Joe give her?	A + ☐ = C	Change Unknown
4. Polly had A cookies. She gave some to Joe. Now she has C cookies. How many cookies did she give to Joe?	A - ☐ = C	Change Unknown
5. Polly had some cookies. Joe gave her B more cookies. Now she has C cookies. How many cookies did she have to start with?	☐ + B = C	Start Unknown
6. Polly had some cookies. She gave B cookies to Joe. Now she has C cookies. How many cookies did she have to start with?	☐ - B = C	Start Unknown

Table 3

Verbal Problem Types with Corresponding Concrete Modeling Strategies

Problem Type	Concrete Modeling Strategy
Change 1 Combine 1	Counting Forward[a] Both given sets are concretely represented and items are counted to obtain the answer.
Change 2 Combine 2	Separating From The larger given set is concretely represented and the smaller given set is separated from this larger set; the remaining set is counted to determine the answer.
Change 3	Adding On The smaller given set is concretely represented and objects are added on to this original set until the larger given set is represented; the number of objects added on are counted to determine the answer.
Change 4	Separating To The larger given set is concretely represented and objects are separated until the smaller given set is reached; the number of objects separated determine the answer.
Change 5 Change 6	Trial and Error A trial number of items is initially represented and objects are added to or separated from this initial set. The initial set is adjusted according to the result obtained. The answer is the number of objects in the final adjusted set.
Equalize Compare	Matching Both given sets are concretely represented and matched in a one-to-one correspondence; the answer is the number of items not in correspondence.

[a] The Counting Forward category is an inclusive term that contains the addition strategies of Counting All, Counting On From First, and Counting On From Larger.

Table 4:. Symbolic Representation of Change and Combine Verbal Problems by First-Grade Children

Study	Verbal Problem Type							
	Change						Combine	
	1	2	3	4	5	6	1	2
Lindvall & Ibarra (1980b)			68*	44	60	47	64	
Carpenter et al. (1983)		91					91	74
Moser & Carpenter (1982)	100	100	75			75		100
Carpenter et al. (1985)	100[a]	100[a]	45[a]	54	64	27	91	45
DeCorte & Verschaffel (1983)	85	78	87			62	87	75

*percent correct

[a]Instructed problem types

Table 5: Correct Solution Percentages for Open Sentence Forms

Study	Grade Level	Open Sentence Form #1 A + B = □	#2 A - B = □	#3 A + □ = C	#4 A - □ = C	#5 □ + B = C	#6 □ - B = C
Weaver (1973)	1	65	55	53	50	46	11
	2	87	80	79	79	76	27
	3	93	88	92	90	88	33
Grouws (1972)	3			65	62	60	37
Blume (1981)	K	71	50	38			
	1	76	69	59			

Table 6

Summary of Instructional Steps for Each Verbal Problem Type

	Type of Representation	Type of Solution
(1) Verbal Problem →	Concrete Model →	Manipulative
(2) Verbal Problem →	Concrete Model ↓ Number Sentence →	Manipulative, Symbolic
(3) Verbal Problem →	Number Sentence →	Manipulative, Symbolic
(4) Verbal Problem ←	Number Sentence	

Table 7

Order of Instructional Presentation of Verbal Problem Types

Verbal Problem	Number Sentence Type	Number Sentence Form
(1) Change 1 & 2	Result Unknown	$A \pm B = \square$
(2) Combine 1 & 2	Result Unknown	$A \pm B = \square$
(3) Change 3 & 4	Change Unknown	$A \pm \square = C$
(4) Change 5 & 6	Start Unknown	$\square \pm B = C$

Table 8

Preinstructional Group Test - Verbal Problems

A. Sam had 8 books.
His sister gave him 7 more books.
Then how many books did Sam have?

B. There were 13 dogs in the park.
9 dogs ran away.
How many dogs were left in the park?

C. Tracy has 6 red balls and 8 green balls.
How many balls does Tracy have in all?

D. There are 11 kittens.
5 of them are brown and the rest are white.
How many white kittens are there?

E. Polly had 9 cookies.
Joe gave her some more cookies.
Then she had 12 cookies.
How many cookies did Joe give her?

F. Kate had 14 dimes.
She lost some of them.
Then she had 9 dimes.
How many dimes did Kate lose?

G. Fred had some stickers.
His aunt gave him 6 more stickers.
Then he had 13 stickers.
How many stickers did Fred have at the start?

H. Joan had some candies.
She ate 4 of them.
Then she had 8 candies.
How many did Joan have to start with?

I. Ted has 4 toy boats.
Sue has 11 toy boats.
How many more boats does Sue have then Ted?

J. Ann found 8 pennies.
Bob found 13 pennies.
How many more pennies does Ann have to find to have as many as Bob?

Table 9

Postinstructional Group Test - Verbal Problems

A. Sue had 9 stickers.
Her brother gave her 4 more stickers.
Then how many stickers did Sue have?

B. Tom had 7 apples.
Liz gave him some more apples.
Then he had 11 apples.
How many apples did Liz give him?

C. Karen had some cookies.
She ate 5 of them.
Then she had 8 cookies.
How many did Karen have to start with?

D. There were 12 cats in the barn.
9 cats ran out.
How many cats were left in the barn?

E. There are 14 puppies.
5 of them are brown and the rest are white.
How many white puppies are there?

F. Ken had 15 pennies.
He lost some of them.
Then he had 8 pennies.
How many pennies did Ken lose?

G. Stacy has 6 blue bows and 7 red bows.
How many bows does Stacy have in all?

H. Dick had some books.
His uncle gave him 5 more books.
Then he had 11 books.
How many books did Dick have at the start?

I. Fred has 8 toy boats.
Sally has 14 toy boats.
How many more boats does Sally have than Fred?

J. Andy found 5 rocks.
Bill found 12 rocks.
How many more rocks does Andy have to find to have as many as Bill?

ble 10: Preinstructional and Postinstructional Performance on Group Tests for Total Sample (N=45) - Change Problems

erbal roblem ype		Correct Sentence	Correct Solution	Direct Representation Form	Direct Representation Complete	Direct Representation Incomplete	Rerepresentation Form	Rerepresentation Complete	Rerepresentation Incomplete	Other
ange 1	pre	30	28	8+7=□	30	6				9
	post	44	40	9+4=□	44	-				1
ange 2	pre	32	29	13-9=□	32	7				6
	post	41	36	12-9=□	41	4				-
ange 3	pre	3	14	9+□=12	--	10	12-9=□	3	-	32
	post	39	38	7+□=11	39	1	11-7=□	-	-	5
ange 4	pre	19	26	14-□=9	--	6	14-9=□	19	8	12
	post	44	30	15-□=8	35	-	15-8=□	9	-	1
ange 5	pre	1	9	□+6=13	--	9	13-6=□	1	-	35
	post	41	35	□+5=11	41	1	11-5=□	-	-	3
ange 6	pre	9	17	□-4=8	--	7	4+8=□	9	2	27
	post	40	28	□-5=8	40	-	5+8=□	-	-	5

Table 11 Preinstructional and Postinstructional Performance on Group Tests for Total Sample (N=45) - Combine, Compare, and Equalize Problems

Verbal Problem Type	Correct Sentence	Correct Solution	Direct Representation Form	Direct Representation Complete	Direct Representation Incomplete	Rerepresentation Form	Rerepresentation Complete	Rerepresentation Incomplete	Other
Combine 1									
pre	31	31	6+8=□	31	6				8
post	41	37	6+7=□	41	1				3
Combine 2									
pre	10	12	5+□=11	-	2	11-5=□	10	-	33
post	35	34	5+□=14	2	-	14-5=□	32	1	9
			14-□=5	1	-				
Compare									
pre	4	10	4+□=11	-	4	11-4=□	4	-	37
post	5	23	8+□=14	3	5	14-8=□	2	-	35
Equalize									
pre	1	15	8+□=13	-	5	13-8=□	1	-	39
post	8	32	5+□=12	7	4	12-5=□	1	-	33

Table 12 Symbolic Solution Pretest-Posttest Comparison (maximum possible correct = 8)

	Mean	Standard Deviation	t-value
Pretest	3.69	2.03	8.2876*
Posttest	6.11	1.80	

*p < 0.05

Table 13 Symbolic Representation Pretest-Posttest Comparison (maximum possible correct = 8)

	Mean	Standard Deviation	t-value
Pretest	2.91	1.71	14.5505*
Posttest	7.16	1.26	

*p < 0.05